ARE PARALLEL ROBOTS MORE ACCURATE THAN SERIAL ROBOTS?

Sébastien Briot and Ilian A. Bonev
Département de génie de la production automatisée, École de technologie supérieure (ÉTS),
1100, rue Notre-Dame Ouest, Montréal, Québec, H3C 1K3, Canada
e-mail: ilian.bonev@etsmtl.ca

Received August 2007

ABSTRACT

It is widely claimed that parallel robots are intrinsically more accurate than serial robots because their errors are averaged instead of added cumulatively, an assertion which has not been properly addressed in the literature. This paper addresses this void by comparing the kinematic accuracy of two pairs of serial-parallel 2-DOF planar robots. Only input errors are considered and all robots are optimized for accuracy, the only constraint being that they cover a given desired workspace. The results of this comparison seem to confirm that parallel robots are less sensitive to input errors than serial robots. However, this comparison is too limited to draw any general conclusions. Besides, it is virtually impossible to make a meaningful comparison between other pairs of serial and parallel robot. Therefore, there is no simple answer to this question of superiority.

EST-CE QUE LES ROBOTS PARALLÈLES SONT PLUS PRÉCIS QUE LES ROBOTS SÉRIELS ?

RÉSUMÉ

Il est généralement dit que les robots parallèles sont intrinsèquement plus précis que les robots sériels parce que leurs erreurs sont moyennées au lieu d'être ajoutées. Cependant, cette hypothèse n'a jamais été vérifiée dans la littérature. Cet article cherche à répondre à cette question en comparant la précision cinématique de deux paires de robots sériels et parallèles à 2 degrés de liberté. Seules les erreurs sur la commande sont prises en compte et tous les robots sont optimisés afin de les rendre les plus précis possible, la seule contrainte étant qu'ils couvrent tous le même espace de travail. Les résultats de cette comparaison semblent confirmer le fait que les robots parallèles sont moins sensibles aux erreurs sur la commande que les robots sériels. Cependant, cette comparaison est trop simpliste pour permettre à tirer des conclusions générales. De plus, il n'est pas possible de comparer d'autres paires de robots sériels et parallèles. Ainsi, il n'y a pas de réponse simple à cette question de supériorité.

Introduction

The development of parallel robots has always been driven by promises of (1) greater rigidity, (2) higher speed, and (3) higher accuracy than serial robots.

The fact that virtually all the hundreds, or even thousands, of motion simulators with load capacities of up to several tons are based on parallel robots (mostly hexapods), with serial robots able to carry at most five hundred kilograms or so, unquestionably demonstrates that the first promise has been fulfilled. The commercial success of the Delta parallel robot and the performance of the recently launched Quickplacer by Fatronik (200 cycles per minute) confirms fulfillment of the second promise, though serial robots are not far behind. But has the third promise been fulfilled yet?

The boom in the development of parallel kinematic machines (PKMs) in the 1990s, particularly those based on hexapods, was driven mainly by that third promise. But none of these hexapods is more accurate than a conventional serial machine tool. Some three-axis and five-axis PKMs are now gaining commercial success, but precision is still not their best feature. While a number of alignment stages are based on parallel robots, the fact remains that great precision is attained by the use of special technologies, such as flexures. Flexures rely on deformation of material to achieve a motion between two elastically joined parts. Flexures are mainly used as passive joints, thus mostly in parallel robots, so it could be said that parallel robots are more accurate for this reason alone. But is it true that parallel robots are *kinematically* more accurate than serial robots because errors are averaged instead of added cumulatively, as widely claimed:

- “The parallel actuator technology promises to offer […] advantages relative to conventional machine tools, such as […] higher accuracy…” [1];
- “Parallel manipulators are preferred to serial manipulators for their […] high positioning accuracy.” [2];
- “Comparing [*sic*] to the traditional serial-chain mechanism […], the parallel mechanism exhibits the following advantages: […] better accuracy due to non-cumulative joint error.” [3] ;
- “The errors of parallel manipulators are averaged out in the serial chains and the errors of serial manipulator are accumulated [*sic*].” [4];
- “Moreover the links [of a serial robot] magnify errors: a small measurement error in the internal sensors of the first one or two links will quickly lead to a large error in the position of the end effector. […] The errors of the internal sensors [of a parallel robot] only slightly affect errors on the platform position.” [5].

Obviously, the sources of positioning error are numerous (design errors, flexibility of the links, thermal expansion, etc.). But, according to Merlet [6], joint sensor errors are the largest source of error in the positioning of a robot.

Surprisingly, we have found no reference that explains this “accumulation/averaging of errors” or which compares the input error sensitivity of serial and parallel robots. The most relevant work was reported in [7], where several two-degrees-of-freedom (2-DOF) planar serial and parallel robots are compared on the basis of four performance criteria, none of which is purely input error sensitivity (one is called sensitivity, but this takes into account errors in the design parameters, in addition to input errors). Our paper addresses this void and provides some new results regarding the input error sensitivity of serial and parallel robots.

In this work, we perform a comparative study of the kinematic accuracy of two serial and two parallel 2-DOF planar robots, one of which was not considered in [7]. The only source of error that we consider is that caused by an uncertainty on the input joint sensor measurement (input errors). The robots are compared in pairs, the robots in each pair being subject to identical actuation (and the same input errors). To make the comparison meaningful, all robots are optimized to have the best accuracy, while covering the same desired square workspace area.

In the next section, we will define the four robots and specify the criteria for comparison. In the third section, we will study the maximal position error of each robot, and, in the fourth section, compare the dexterity index to the maximal position error. Conclusions will be presented in the last section.

1. Criteria for Comparison and Description of the Planar Robots Under Study

In most cases, the so-called *dexterity index* is used to study the kinematic accuracy of robots [8]. Merlet [9] criticizes such an index, stressing that its major drawbacks are that it mixes both translational and rotational terms of the Jacobian matrix and that it is usually not invariant on the choice of units. As a consequence, the Jacobian matrix must be split into its translational and rotational parts to calculate the dexterity of each of them, but this is not satisfactory for estimating the amplification factor for motion involving both translational and rotational displacements. As we will see in this paper, the dexterity index does not even work properly for robots having only translational degrees of freedom. Thus, the most suitable method for computing the accuracy of robots (actually the input error sensitivity) is to calculate the *maximal position error,* or *orientation error*, due to input errors, at a given nominal configuration. This is very easy to do for 2-DOF planar parallel robots using a simple geometrical method.

Now that we have decided how to measure the kinematic accuracy of robots, we have to define some criteria for a fair comparison. The first—and most obvious—criterion is that the robots must have the same actuators (only revolute or only prismatic) so that they can have the same input errors. Another criterion is that the robots must be able to accomplish the same task. We impose the constraint in this paper that the robots must be able to displace their end-effectors inside a 1 m by 1 m square (the *desired workspace*), and do so with the best accuracy possible, meaning that their designs should be optimized to have the smallest mean maximal position error over this desired workspace. This square should obviously be free of singularities. Note that the authors of [7] do not compare robots with optimized kinematic accuracy.

That said, we will compare the following two pairs of 2-DOF planar robots for positioning:

- a *RRRRR* parallel robot (Fig. 1a) and a *RR* serial robot (Fig. 1b);
- a *PRRRP* parallel robot, the directions of its base-mounted prismatic actuators being parallel (Fig. 1c), and a Cartesian serial robot (Fig. 1d).

While the choice of the first pair is fairly obvious, the choice of the second pair might look a bit arbitrary, but it is not. We choose a *PP* serial robot in which the directions of the prismatic joints are orthogonal, simply because any other non-Cartesian *PP* serial robot will have worse maximal position error. As for the *PRRRP* parallel robot, of course, we could choose another architecture with prismatic actuators, but this one is surely the most practical one. Finally, we choose to have the directions of its two prismatic actuators parallel, simply because this gives the most compact design having the desired singularity-free workspace.

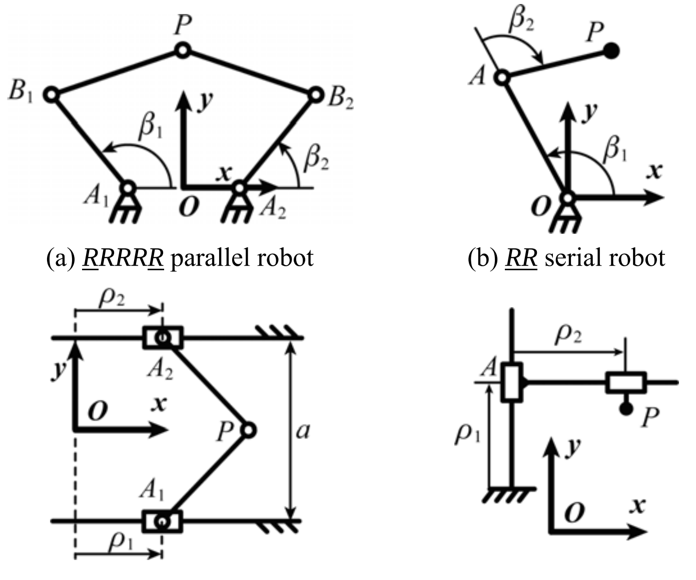


Fig. 1. The two pairs of planar robots under comparison (not to scale).

Both serial robots are well known and trivial to design to obtain the best accuracy within the desired workspace.

The <u>*RR*</u> serial robot is designed by finding the optimal values of the parameters *OA*, *AP* and *d* (Fig. 2a). For this robot, it is obvious that the smaller the workspace, the higher the accuracy. Therefore, the design parameters have to define a compact workspace with respect to the desired workspace. The geometric conditions for compactness are:

$$OA + AP = \sqrt{(1+2\gamma+d)^2 + (O.5+\gamma)^2}\ , \tag{1}$$

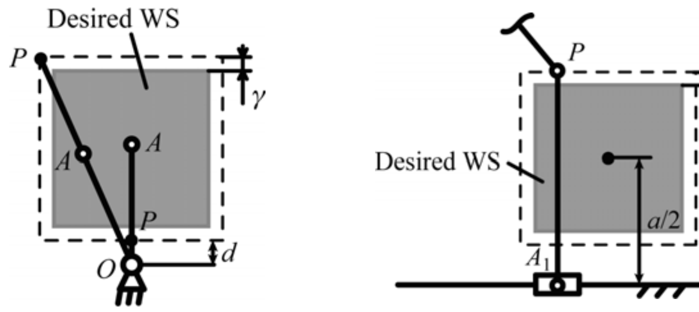
$$|OA - AP| = d\ . \tag{2}$$

where γ is a safety distance added to avoid that the desired workspace includes singularities along its boundary. In this study, $\gamma = 0.1$ m.

Solving equations (1) and (2), we obtain the possible values for *OA* and *AP* as functions of *d*:

$$\begin{cases} OA = (T+d)/2 \\ AP = (T-d)/2 \end{cases} \text{ or } \begin{cases} OA = (T-d)/2 \\ AP = (T+d)/2 \end{cases} \tag{3}$$

where $T = \sqrt{(1+2\gamma+d)^2 + (1/2+\gamma)^2}$.



(a) *RR* serial robot (b) *PRRRP* parallel robot

Fig. 2. Design constraints for obtaining the desired workspace.

The mean value of the maximal position error within the desired workspace of the *RR* serial robots corresponding to any of the two solutions of eq. (3) is shown in Fig. 3a, as a function of d. The calculation of the maximal position error will be presented in Section 3. As expected, the optimal design occurs at $d = 0$ m, and from eq. (3), we have $OA = 0.67$ m and $AP = 0.67$ m.

The accuracy of the Cartesian serial robot is the same for any position and any actuator stroke. Therefore, there are no optimal design parameters to look for.

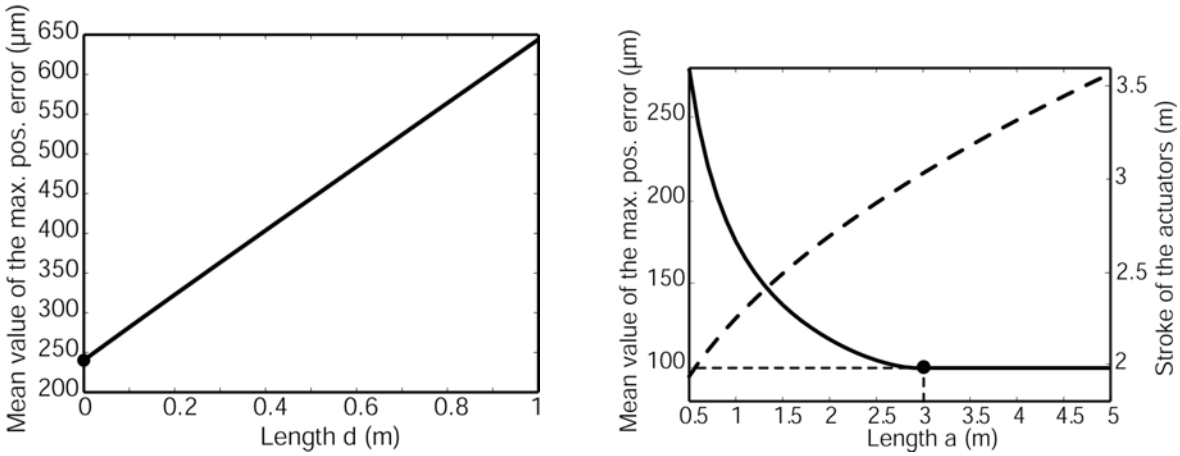
The two parallel robots are more difficult to optimize in terms of accuracy. These difficulties are due to the complexity of their direct kinematics and to the presence of singularities inside their workspaces. These two robots have recently been studied in detail [10–12]. In these references, the authors analyze the robots using different performance indices depending on the link lengths. From [11], we can roughly estimate that a nearly-optimal *RRRRR* design occurs when $A_1A_2 = 0.3$ m, $A_1B_1 = 0.6$ m, and $B_1P = 0.8$ m. Although this is not the actual optimal design, for the purposes of our purely qualitative study, it will be good enough. What is important is that the *RR* serial robot has been given all the chances to win the competition—its design is optimal.

For the *PRRRP* parallel robot, the shorter the links A_iP, the higher the accuracy. While this fact seems obvious, it was nevertheless verified numerically. Thus, it is possible to find a relationship between a and A_iP which defines the minimal link length as (Fig. 2b):

$$A_iP = (a+1)/2 + \gamma \, . \tag{4}$$

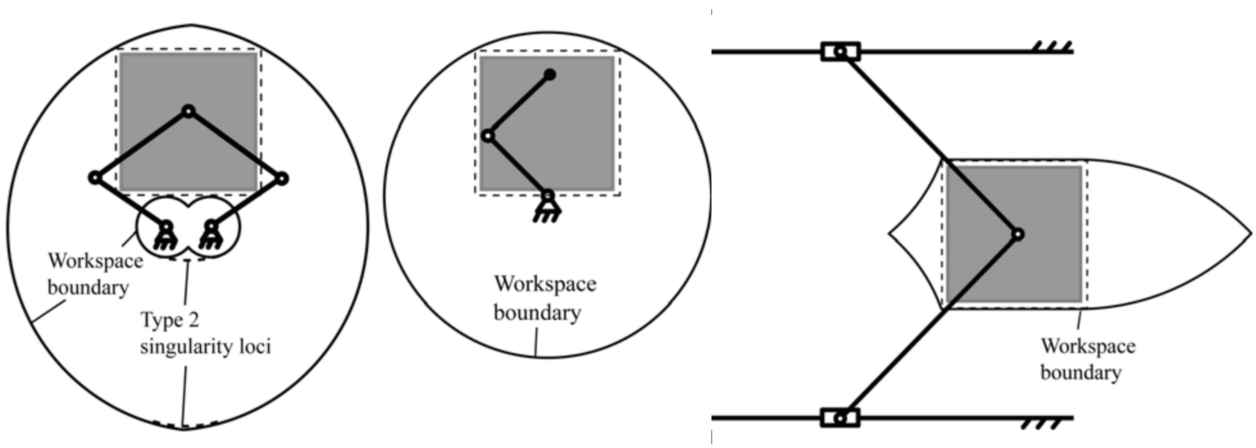
The mean value of the maximal position error within the desired workspace of the *PRRRP* parallel robots whose parameters obey eq. (4) is shown in Fig. 3b, as a function of the parameter a. The method for calculating the maximal position error will be presented in Section 3. Figure 3b also shows the required stroke of the actuators (in dashed line). One can see that higher accuracy calls for longer actuators. Thus, we chose a nearly-optimal design at $A_1P = A_2P = 2.1$ m and $a = 3$ m.

Figure 4 shows the optimized designs of three of the robots under study, their workspaces and the square within them that constitutes the desired workspace, all to the same scale. The workspace of the Cartesian serial robot, which is not shown here, is obviously a rectangular region.



(a) *RR* serial robot (b) *PRRRP* parallel robot

Fig. 3. Variations of the mean value of the maximal position error over the desired workspace.



(a) *RRRRR* parallel robot (b) *RR* serial robot (c) *PRRRP* parallel robot

Fig. 4. Workspace of the planar robots under study (to scale).

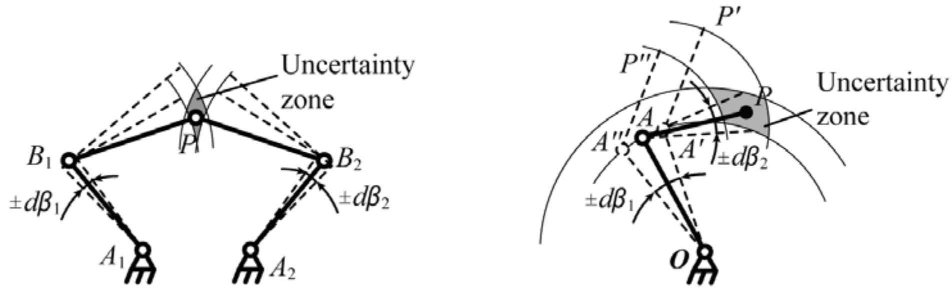
In the next section, we will analyze the maximal position errors of these four robots using a geometrical method.

2. Analysis of the Maximal Position Error of the Robots Under Study

3.1. Comparison of the *RRRRR* parallel robot and the *RR* serial robot

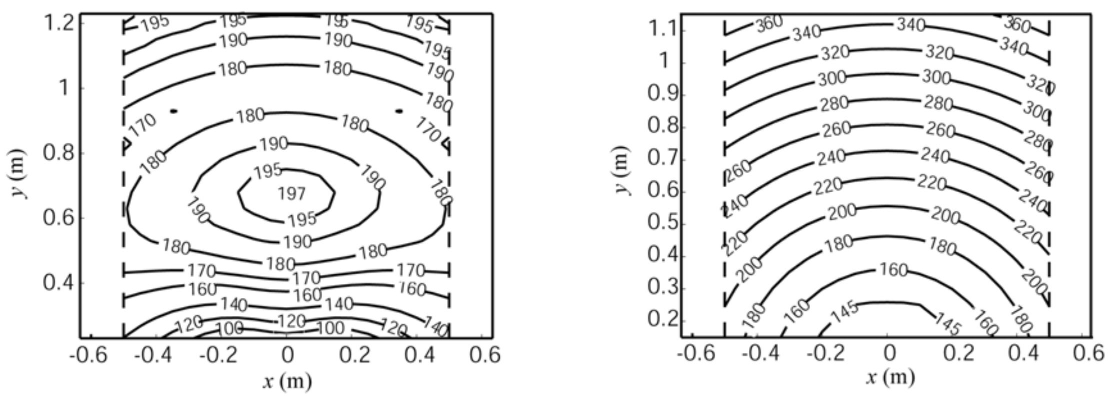
For these robots, we consider that the maximal input error is equal to $\pm 2\times 10^{-4}$ rad. The maximal position error for these robots is quite easy to determine. For each of them, this error occurs at one of the four sets of extreme input errors, i.e., at one of the corners of the so-called uncertainty zone, as shown in Fig. 5.

Thus, it is possible to calculate the maximal position error at each nominal position. These errors are presented in Fig. 6. In addition, Table I gives some statistics regarding the maximal position error for each robot over the desired workspace.



(a) *RRRRR* parallel robot (b) *RR* serial robot

Fig. 5. Uncertainty zones for the first two planar robots.



(a) *RRRRR* parallel robot (b) *RR* serial robot

Fig. 6. Maximal position errors (in µm) in the desired workspace for the first two planar robots.

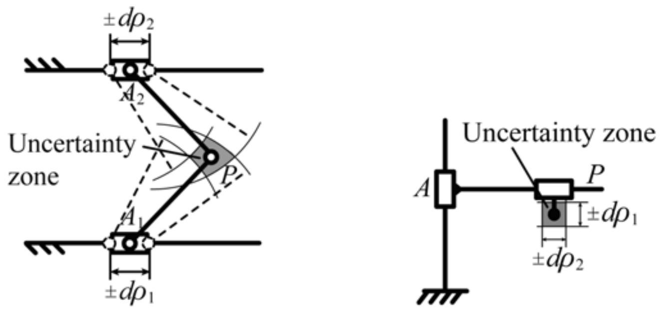
TABLE I
Statistics for the Maximal Position Errors

Max. position error	*RRRRR* parallel robot	*RR* serial robot
Max. value (µm)	197	367
Mean value (µm)	175	240
Standard deviation (µm)	24	61

3.2. Comparison of the *PRRRP* parallel robot and the Cartesian serial robot

For these robots, we consider that the maximal input error is equal to ±100 µm. The maximal position error for these robots is also quite easy to determine. It is equal to approximately 141 µm for the Cartesian serial robot (Fig. 7b). For the *PRRRP* parallel robot, this error occurs at one of the four sets of extreme input errors, i.e., at one of the corners of the so-called *uncertainty zone*, as shown in Fig. 7a.

Thus, it is possible to obtain the maximal position error at each position for the *PRRRP* parallel robot. This maximal position error is virtually equal to the input error, i.e., ±100 µm, for any position. Therefore, no contour plot as in Fig. 6 is given for this robot. Table II gives statistics regarding the maximal position error for each robot over the desired workspace.



(a) *PRRRP* parallel robot (b) Cartesian serial robot

Fig. 7. Uncertainty zones for the second pair of planar robots.

TABLE II
Statistics for the Maximal Position Errors

Max. position error	*PRRRP* parallel robot	*PP* serial robot
Max. value (μm)	100	141
Mean value (μm)	100	141
Standard deviation (μm)	0	0

In concluding this section, we warn readers that our study is too limited to draw any general conclusions. It is quite possible, for example, that if the desired workspace is different (e.g., an annular region or an elongated rectangular region) or if the input errors are much smaller, some of the results could be quite different.

Nevertheless, our study suggests that a *RRRRR* parallel robot is much less sensitive to input errors than an equivalent *RR* serial robot, while having nearly the same overall dimensions. (We again point out that by "equivalent" we mean that the robots have the same desired square workspace and the same input errors.) Similarly, a *PRRRP* parallel robot is much less sensitive to input errors than an equivalent Cartesian serial robot, but only when its overall dimensions are much greater than those of the serial robot. In fact, the mean maximal position error of a nearly-optimal *PRRRP* parallel robot is equal to its maximal input error, which means that both the Cartesian robot and the parallel robot are dimension invariant; hence the comparison is fair. However, with greater dimensions there are more manufacturing errors, and a need for larger actuators stroke, which means higher manufacturing costs. For example, the actuators of the *PRRRP* parallel robot should be three times as long as those of the Cartesian robot. Thus, we find it hard to believe that, in practice, a *PRRRP* parallel robot would be more precise than a Cartesian robot.

Finally, we are tempted to comment on this widely claimed "accumulation of errors" in serial robots in contrast to the "averaging of errors" in parallel robots. A ±100 μm input error produces a maximal position error of approximately 141 μm in a Cartesian serial robot, and a maximal mean positioning error of about 100 μm in a (optimally designed) *PRRRP* parallel robot. So, in this example, one might indeed say that there is an averaging of errors in the parallel robot. However, when its end-effector is close to certain singularities, then the maximal position error could be several times larger than the input error (in the case of prismatic actuators). We therefore believe that this is, in general, too strong a statement and should be avoided.

3. Analysis of the Dexterity Index

In this section, we will compare the dexterity indices of the four robots under study to their maximal position errors over the desired workspace. The dexterity of a robot is calculated as defined in [9]:

$$\xi = \frac{1}{\|\mathbf{J}\|\|\mathbf{J}^{-1}\|}, \tag{5}$$

where we use the Euclidean norm defined as

$$\|\mathbf{J}\| = \sqrt{tr\left(\frac{1}{2}\mathbf{J}\mathbf{J}^{T}\right)}, \tag{6}$$

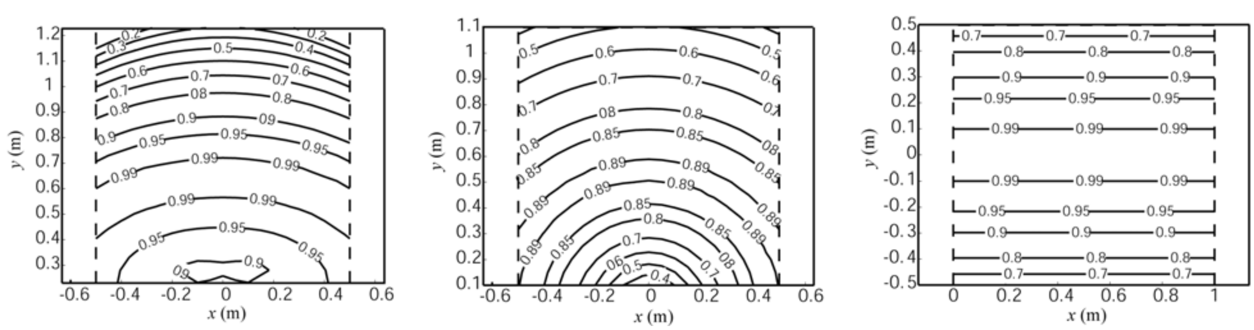
and **J** is the Jacobian matrix of the robot.

The Jacobian matrices for the four robots under study will not be derived here, since this is fairly simple to do and the calculation of the dexterity index is not the main subject of this paper.

The dexterity maps for the two parallel robots and for the *RR* serial robot are represented in Fig. 8. The dexterity index of the Cartesian serial robot is constant and equal to unity, and is therefore not shown in this figure.

Several observations can be made. Firstly, the shape of the dexterity map does not correspond to the shape of the maximal position error map. Areas of highest dexterity do not correspond to areas of lowest maximal position error. Secondly, the values of the dexterity indices of the *RRRRR* parallel robot and of the *RR* serial robots are similar, even though the first robot is more accurate than the second in terms of maximal position error. Likewise, the values of the dexterity index for the *PRRRP* parallel robot are not constant, even though its maximal position error is nearly constant.

It is therefore clear that the dexterity map cannot be used to evaluate the global nature of the accuracy of a robot, let alone to compare the accuracy of different robot designs, and this is true even for robots having only positioning capabilities (i.e., when there are no problems involving mixed units).



(a) *RRRRR* parallel robot (b) *RR* serial robot (c) *PRRRP* parallel robot

Fig. 8. Contour plots of the dexterity index for three of the robots over the desired workspace.

4. Conclusions

Obviously, our study is quite simplistic and too limited for us to draw any general conclusions as numerous other authors have done who claim that parallel robots are more accurate than serial robots. Yet, in spite of the limited nature of our study, it still suggests that parallel robots are indeed *theoretically* more accurate than serial robots, when input errors are assumed to be the only source of inaccuracy. This was shown using the natural concept of maximal position error instead of the dexterity index, which proved to be meaningless for comparing accuracy.

But, are parallel robots more accurate than serial robots in practice, where the larger dimensions required in parallel robots might induce greater mechanical errors? Moreover, are we not, after all, comparing apples and oranges when we state that parallel robots are more accurate than serial robots? How can we say, for example, that a hexapod (with six linear actuators) is theoretically more accurate than a serial machine tool (with three linear and three rotary actuators)? The truth is that only practice, and not theory, will show whether or not parallel robots can be *manufactured* to be more accurate than serial robots. In other words, we believe that the mechanical design of a robot, its manufacture and its calibration are much more important drivers of accuracy than the optimal kinematic design. So, the question is whether parallel robots can be mechanically designed, manufactured and calibrated so as to be more precise than the most accurate serial robots on the market. For the time being, we can only claim that some parallel robots seem to be less sensitive to input errors than their equivalent serial counterparts.

5. References

[1] A. J. Wavering, “Parallel kinematic machine research at NIST: past, present and future,” in *Parallel Kinematic Machines*, Advanced Manufacturing Series, Springer, pp. 17–31, 1999.

[2] A. Rauf, S.-G. Kim, and J. Ryu, “A new measurement device for complete parameter identification of parallel manipulators with partial pose measurements,” *The 4th Chemnitz Parallel Kinematics Seminar*, Chemnitz, Germany, pp. 89–106, 2004.

[3] J. Song, J.-I Mou, and C. King, “Error modeling and compensation for parallel kinematic machines,” in *Parallel Kinematic Machines*, Advanced Manufacturing Series, Springer, London, pp. 171–187, 1999.

[4] L. Guan, Y. Yun, J. Wang and L. Wang, “Kinematics of a Tricept-like parallel robot,” *2004 IEEE International Conference on Systems, Man and Cybernetics*, pp. 5312–5316, October 10–13, 2004.

[5] J.-P. Merlet, Parallel Robots, 2nd ed., Springer, pp. 4–7, 2006.

[6] J. P. Merlet, “Computing the worst case accuracy of a PKM over a workspace or a trajectory,” *The 5th Parallel Kinematics Seminar*, Chemnitz, Germany, pp. 83–95, 2006.

[7] P. Wenger, C. Gosselin, and B. Maillé, “A comparative study of serial and parallel mechanism topologies for machine tools,” *International Workshop on Parallel Kinematic Machines*, Milan, Italy, pp. 25–35, 1999.

[8] C. Gosselin, “The optimum design of robotic manipulators using dexterity indices,” *Robotics and Autonomous Systems*, Vol. 9, No. 4, pp. 213–226, 1992.

[9] J.-P. Merlet, “Jacobian, manipulability, condition number, and accuracy of parallel robots,” *Journal of Mechanical Design*, Vol. 128, No. 1, pp. 199–206, 2006.

[10] X.-J. Liu, J. Wang, and G. Pritschow, "Kinematics, singularity and workspace of planar 5R symmetrical parallel mechanisms," *Mechanism and Machine Theory*, Vol. 41, No. 2, pp. 145–169, 2006.
[11] X.-J. Liu, J. Wang, and G. Pritschow, "Performance atlases and optimum design of planar 5R symmetrical parallel mechanisms," *Mechanism and Machine Theory*, Vol. 41, No. 2, pp. 119–144, 2006.
[12] X.-J. Liu, J. Wang, and G. Pritschow, "On the optimal kinematic design of the PRRRP 2-DOF parallel mechanism," *Mechanism and Machine Theory*, Vol. 41, No. 9, pp. 1111–1130, 2006.